AB Calculus – summer work

NO calculators *are permitted in doing this worksheet. It is best if the work is done individually as its intent is to review concepts and skills that are necessary for calculus. This worksheet should be ready to be handed in the FIRST CLASS MEETING !!!! The* ***VERY*** *first class meeting.*

You are also expected to know better than your best friend's instagram the following:
THE UNIT CIRCLE LOG RULES DOMAINS and RANGES OF COMMON PARENT FUNCTIONS

1. Solve the following for *x* in terms of *y*.

(a) $y = \ln(x+1)$

(b) $y = \sin x$

(c) $y = e^x - 2$

(d) $y = \dfrac{1}{x^2+2}$

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2. Evaluate the following limits:

(a) $\lim\limits_{x\to 3} (2x+1)$

(b) $\lim\limits_{x\to 2} \dfrac{x-2}{x^2-4}$

(c) $\lim\limits_{x\to\infty} \dfrac{\sin x}{x}$

(d) $\lim\limits_{x\to 0} \dfrac{\sin 6x}{x}$

(e) $\lim\limits_{x\to\infty} \dfrac{x^4+3}{2x^2-5}$

(f) $\lim\limits_{x\to\infty} \cos\left(\dfrac{3\pi}{x}\right)$

#2 continued

(g) $\lim_{x \to 3^+} \dfrac{x-5}{x-3}$

(h) $\lim_{x \to 7} \dfrac{\sqrt{x+2}-3}{x-7}$

3. Consider the piecewise function defined below.

$$f(x) = \begin{cases} x^2 + 1 & x < 2 \\ k & x = 2 \\ 3x - 1 & x > 2 \end{cases}$$

(a) Evaluate : $f(0) =$ $f(1) =$ $f(3) =$

(b) Determine $\lim_{x \to 2} f(x)$. Show the analysis that leads to your answer.

(c) Determine the value of *k* so that $f(x)$ is defined and continuous at $x = 2$.

(d) Write a piecewise function for $f'(x)$ (only for the top and bottom expressions)

4. Solve the following inequalities using an interval line analysis

(a) $(x-3)^2(x+2)<0$

(b) $\dfrac{(2x+1)(x-1)}{(x-2)(x+5)^2}\geq 0$

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5. Determine the derivatives of the following functions

(a) $f(x)=\sqrt{x}+x^{-1}-3x^2$

(b) $g(x)=x^3-2x^2+5$

(c) $h(x)=x^3-\dfrac{3}{x^2}+6-2e^x$

(d) $k(x)=3\sin x-2\cot x+\ln x$

6. Simplify the following complex fraction: $\dfrac{\sqrt{x+3}-\dfrac{x}{\sqrt{x+3}}}{(x+3)}$

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7. Consider the function $f(x)=-x^2+3x+4$. Determine:

a. Determine an expression for $f'(x)$.

b. Find the slope of the tangent to the curve of $f(x)$ when $x=2$.

c. Write an equation of the line tangent to $f(x)$ at $x=2$.

d. Write an equation of the normal line at the point $(2, f(2))$

e. Evaluate the average rate of change of $f(x)$ over $[0, 4]$.

f. Determine the point on the graph of f where the instantaneous rate of change is -5.

#7 continued

g. Determine the *x*-coordinate on the curve where the normal line is parallel to $x + 2y = 7$.

h. Write an equation of the horizontal tangent to $f(x)$.

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8. A particle moves along a straight line so that its position at any time $t \geq 0$ can be determined by the function $s(t) = t^3 - 3t^2 - 5$.

(a) Determine functions for the velocity and the acceleration of the particle at any time t, $t \geq 0$

(b) Find the velocity of the particle when the acceleration is zero.

(c) Find the position of the particle when the velocity is zero.

(d) Determine the displacement of the particle between stopping times.

(e) Determine the interval when the particle is moving right and speeding up.

9. Consider the function $f(x) = 2x^{\frac{3}{2}} - 6x$.

(a) Write an equation of the line tangent to the curve at $x = 1$.

(b) Write equations for of all horizontal tangents to the curve.

(c) Evaluate $f''(1)$.

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10. Given $f(x) = 2\pi \sin x + \pi x$

(a) Determine the point(s) on the curve on the interval $[0, 2\pi]$ where the normal line is vertical.

(b) Write an equation of the line tangent to the curve at $x = 0$.